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Vortices and Angular Momentum in Bose-Einstein-Condensed Cold Dark Matter Halos

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Abstract. If cold dark matter elementary particles form a Bose-Einstein condensate, their superfluidity may distinguish them from other forms of cold dark matter, including creation of quantum vortices. We demonstrate here that such vortices are favoured in strongly-coupled condensates, while this is not the case for axions, which are generally presumed to form a Bose-Einstein condensate but are effectively non-interacting.

Introduction

Suggestions have appeared in the literature that cold dark matter (CDM) may be in the form of a Bose-Einstein condensate (BEC), including axionic and other forms of CDM. This has important implications for the physics of structure formation, notably at small scales where one expects significant deviations from the more standard CDM, due to the superfluidity exhibited by BECs. In fact, a prime motivation for considering self-interacting BECs for CDM was their ability to produce galactic halos with constant density cores, see e.g. Goodman (2000) and Peebles (2000). The corresponding profiles may then better agree with observed rotation curves of dwarf and LSB galaxies, see e.g. Böhmer & Harko (2007), Böhmer, Martins & Salucci (2009). The inner structure of halos may also depend on the angular momentum distribution of the CDM particles. Laboratory BECs are known to develop vortices when rotated with a sufficient angular velocity. Duffy & Sikivie (2008) report that certain fine-structure in the observed inner mass distribution of the Milky Way can be explained only if the infalling dark matter particles from which such galactic halos formed had a net overall rotation, causing a 'tricusp' caustic ring of dark matter in that case. For standard, non-interacting CDM models, however, one expects infall to be *irrotational*, while Sikivie & Yang (2009) argue that axionic dark matter, as a BEC, may form vortices leading to net overall rotation. As such, they suggest, Milky-Way observations may already have detected the signature of axionic CDM. Vortices have also been postulated for strongly-coupled BECs involving ultralight scalar particles (Silverman & Mallet (2002)). The question of whether an angular velocity sufficient to create vortices occurs in BEC dark matter cosmologies has not yet been answered, however. We address this point here by calculating the critical angular velocity for vortex creation in a simple

model of BEC/CDM galactic halos and comparing the result with the angular velocity expected from cosmological N-body simulations of CDM.

The Model

We shall use an energy argument to derive the critical angular velocity for vortex creation in a rotating, self-gravitating BEC halo by finding the angular velocity above which the energy is lowered by the presence of a vortex. For the unperturbed equilibrium state, we model the BEC halo as an oblate Maclaurin spheroid - a homogeneous ellipsoid of mass density $\rho = mn$, with semi-axes (a, b, c) along (x, y, z) such that $a = b > c$, of total volume $V = 4\pi a^2 c/3$, uniformly rotating with angular velocity $\mathbf{\Omega} = \Omega\hat{\mathbf{z}}$, with gravitational potential $\Phi(r, z) = \pi G\rho\left(A_1(e)r^2 + A_3(e)z^2\right)$ in cylindrical coordinates (r, z). The functions $A_1(e), A_3(e)$ depend on the excentricity e, defined via $e = \sqrt{1-(c/a)^2}$, and there is a family of solutions parameterized by Ω/Ω_G, or, equivalently, e, related to each other according to (see Binney & Tremaine (1987))

$$\left(\frac{\Omega}{\Omega_G}\right)^2 = 2\left[A_1(e) - (1-e^2)A_3(e)\right], \tag{1}$$

where $\Omega_G \equiv \sqrt{\pi G\rho}$ is a characteristic gravitational angular frequency. For $\Omega = 0$, $e = 0$, while e must not exceed 0.9529 (*ibid.*).

We describe these self-gravitating BEC halos of ellipsoidal shape with varying degrees of rotational support by self-consistently coupling the Gross-Pitaevskii (GP) equation of motion for the complex scalar wavefunction $\psi(\mathbf{r}, t)$ to the Poisson equation, where $|\psi|^2(\mathbf{r}, t) = n$, the number density of particles of mass m:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + (m\Phi + g|\psi|^2 - \mu)\psi, \quad \Delta\Phi = 4\pi Gm|\psi|^2. \tag{2}$$

We assume that halos are comprised of N particles in the condensed state described by ψ, and so $\int_{\mathbb{R}^3}|\psi|^2 = N$, which determines the GP chemical potential μ. BEC/CDM, like standard CDM, is assumed to interact so weakly with other matter and radiation, once its abundance is fixed in the early universe, that we can neglect all other, non-gravitational couplings. However, BEC dark matter is *self-interacting*, as described by an effective interaction potential $g|\psi|^4/2$ with coupling constant $g = 4\pi\hbar^2 a_s/m$, where a_s is the 2-body scattering length. [Since BECs with negative scattering length are not stable in the context described here, we consider only $g > 0$.]

In a frame rotating with velocity $\mathbf{\Omega}$, our system is stationary, so the derivative with respect to time in equ.(2) vanishes. The equation of motion in this frame is then given by equ.(2) with an additional operator $V_{rot} = -\mathbf{\Omega}\cdot\mathbf{L}$ on the right-hand-side, where $\mathbf{L} = -i\hbar\mathbf{r}\times\nabla$, and it is also understood that the respective variables and quantities are in the new frame. Incompressible Maclaurin spheroids are approximate solutions of this system of equations.[1] Such station-

[1]Uniform density is a more realistic approximation for BEC halos than for the cuspy halos of standard CDM; equ. (2) - with or without rotation - favours a flat core over the cuspy r^{-1}-profile found by N-body simulations of standard CDM.

ary systems can then be studied via the corresponding GP energy functional, given by

$$\mathcal{E}[\psi] = \int_{\mathbb{R}^3} \left[\frac{\hbar^2}{2m} |\nabla\psi|^2 + \frac{m}{2}\Phi|\psi|^2 + \frac{g}{2}|\psi|^4 + i\hbar\psi^* \mathbf{\Omega} \cdot (\mathbf{r} \times \nabla\psi) \right]. \tag{3}$$

We shall use this equation to determine at which angular velocities the presence of a vortex is energetically favoured. To this aim, we decompose the wave function ψ into a vortex-free part and a part which carries the vorticity, thus splitting the energy functional in equ.(3) so as to compare the respective energy contributions, with or without vortices, more easily. To derive an analytical result with as much generality as possible, we consider the following ansatz. The vorticity-part is a d-quantized straight vortex, modeled as a funnel-shaped tube along the rotation-axis, with core radius s, where the density outside of the core is given by the unperturbed profile of the Maclaurin spheroid, whereas the density in the vortex core region drops to zero at the center.

Results

When our wave function ansatz is inserted in equ.(3), there is a critical angular velocity Ω_c above which the energy is lowered by the presence of a vortex. Since our ansatz leads to an energy greater than or equal to that which would result if the 'real' wave function were used in place of the ansatz, this means vortex creation is energetically favoured, in general, if $\Omega > \Omega_c$. The general expression for Ω_c (for brevity not shown here; for derivation see Rindler-Daller & Shapiro (2010)) depends on the core radius s, which is of the order of the healing length ξ defined by equating the energy contributions from the kinetic term and self-interaction, so $\xi^2 = \hbar^2/(2\rho g)$. In the strongly-coupled regime, as $g \to \infty$, $\xi \to 0$ and $s \to 0$, respectively, and the critical angular velocity has a leading-order term which diverges logarithmically, $\Omega_c \simeq \Omega_{QM} d \ln \frac{a}{s}$, just as for laboratory condensates (see e.g. Lundh et al. (1997)). Here we define $\Omega_{QM} \equiv \hbar/(ma^2)$, the characteristic angular frequency such that every particle contributes an amount $\hbar$ to the total angular momentum of the uniformly rotating Maclaurin spheroid ($|\mathbf{L}| = N\hbar$). On the other hand, if s or ξ are comparable to the system size a, this logarithm is subleading. We note that Ω_c is a monotonically increasing function of the winding number d, so we restrict our consideration to the lowest Ω_c, for which $d = 1$. Then, the core radius can be replaced by the healing length (see Pitaevskii & Stringari (2003)), and the critical angular velocity (in units of Ω_G) becomes

$$\frac{\Omega_c}{\Omega_G} = \frac{\Omega_{QM}}{\Omega_G} \left[\ln\frac{a}{\xi} + \frac{25}{12} + \frac{\pi}{2} \left(\frac{\Omega_G}{\Omega_{QM}} \right)^2 \left(\frac{A_1(e)}{6} \left(\frac{a}{\xi} \right)^{-4} + A_3(e) \frac{1-e^2}{9} \left(\frac{a}{\xi} \right)^{-2} \right) \right]. \tag{4}$$

We determine whether a given set of BEC parameters (m, g) makes the spheroid rotation velocity equal the critical value above which a vortex forms, $\Omega = \Omega_c$, by combining equs.(1) and (4), for a given e. We observe that $\Omega_{QM}/\Omega_G = m_H/m$ and $a/\xi = (g/g_{min})^{1/2}$, where $m_H \equiv \hbar/[a^2(\pi G\rho)^{1/2}]$ and $g_{min} \equiv \hbar^2/(2\rho a^2)$, the latter of which depend only on halo parameters. The solution for a given e

is a curve in the $(m/m_H, g/g_{min})$-plane, for which there is a minimum allowed value of $g/g_{min} \geq 1$. For each curve, no vortex is allowed for parameters in the space above the curve. According to equ.(4), Ω_c goes to infinity as $g \to 0$. This is the case for axion dark matter, for which the coupling is so weak as to be effectively zero.

We determine the e-values of interest for BEC/CDM halos as follows. The superfluidity effects of BEC dark matter which distinguish it dynamically from standard CDM are mostly limited to the internal structure of our halos, while larger-scale structure formation is otherwise the same. The latter is responsible for the tidal torques that give a halo its angular momentum. Cosmological N-body simulations of the CDM universe show that halos form with a net angular momentum such that the dimensionless ratio $\lambda = L|E|^{1/2}/GM^{5/2}$, which expresses their degree of rotational support, has values in the range $[0.01, 0.1]$ with median value 0.05 (see e.g. Barnes & Efstathiou (1987)), where L is the angular momentum, E is the halo binding energy, and M is the total mass. For our spheroids, λ^2 corresponds roughly to the ratio of rotational kinetic energy to gravitational potential energy, and can be expressed in terms of e only,

$$\lambda = \frac{6}{5\sqrt{5}} \frac{\arcsin(e)}{e} \left(\frac{3}{2e^2} - 1 - \frac{3\sqrt{1-e^2}}{2e \arcsin(e)} \right)^{1/2} .$$

In what follows, we take three representative values for λ, $(0.01, 0.05, 0.1)$, which correspond to $e = (0.051, 0.249, 0.464)$, respectively. For given values of the halo mass density and eccentricity, the angular velocity is fixed. For example, a Milky-Way-sized halo with $M = 10^{12} M_\odot$ and a radius of $R = 100$ kpc, where the corresponding Maclaurin spheroid with the same mean mass density has a semi-axis given by $R = a(1-e^2)^{1/6}$, has a density of about 10^{-26} g/cm^3 and $\Omega \sim 10^{-17}$ rad/s for $\lambda = 0.05$, so $\Omega/\Omega_G \sim 0.18$. For such a halo $m_H \sim 10^{-58}$ g and $g_{min} \sim 10^{-76}$ erg cm^3. In Fig.1, we plot m/m_H versus g/g_{min} for the above λ-values, independent of halo size.

For BEC halos, the condition of gravitational equilibrium restricts the values of m and g to another curve in the $(m/m_H, g/g_{min})$-plane. In the strongly-coupled regime, non-rotating BEC halos are just $(n = 1)$-polytropes, for which the size R is related to the BEC parameters according to $R = \pi[\hbar^2 a_s/(Gm^3)]^{1/2}$ (see Böhmer & Harko (2007)), which we translate into our language as

$$1 = \left(\frac{\Omega_{QM}}{\Omega_G} \right)^2 \frac{\pi^2}{8(1-e^2)^{1/3}} \left(\frac{a}{\xi} \right)^2 . \qquad (5)$$

The resulting relationship between m/m_H and g/g_{min} is plotted in Fig.1, as well. For CDM halo λ-values, the degree of rotational support is small enough that equ.(5) should still be a good approximation. There is almost no sensitivity to e, so the respective curves for different e lie on top of each other. Since the BEC parameters which satisfy (5) are all below our critical curves for which $\Omega = \Omega_c$, BEC/CDM halos, in general, *will* typically form vortices.

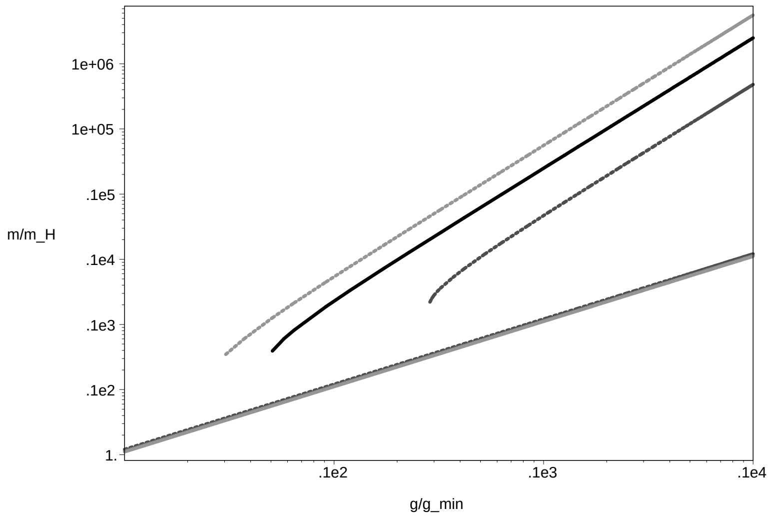

Figure 1. Dimensionless BEC particle mass m/m_H vs. coupling strength g/g_{min}: critical lines $\Omega = \Omega_c$ for $\lambda = 0.01$ ($e = 0.051$) (grey-dotted), $\lambda = 0.05$ ($e = 0.249$) (black-solid), $\lambda = 0.1$ ($e = 0.464$) (light grey-dotted); BEC halo ($n = 1$)-polytropes: lower-most curves (grey-solid) for the same e-values

Conclusions

While axions, with an effectively zero coupling, apparently do not form vortices, vortices *will* be created, in general, for BEC/CDM halos in the strongly-coupled regime. As such, previous BEC models of halo mass profiles, which do not account for their presence, should be revised, especially at the centers where vortex creation causes the density to drop.

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References

Barnes J., Efstathiou G., ApJ **319**, 575, (1987)
Binney J., Tremaine S., *Galactic Dynamics*, Princeton Univ. Press (1987)
Böhmer C.G., Harko T., JCAP **06**, 025, (2007)
Böhmer C.G., Martins C.F., Salucci P., private communication
Duffy L.D., Sikivie P., PRD **78**, 063508 (2008)
Goodman J., New Astronomy **5**, no.2, 103 (2000)
Lundh E., Pethick C.J., Smith H., Phys.Rev. A **55**, 2126 (1997)
Peebles P.J.E., ApJ **534**, 2, L127 (2000)
Pitaevskii L.P., Stringari S., *Bose-Einstein Condensation*, Oxford Science Publ. (2003)
Rindler-Daller T., Shapiro P.R., subm. (2010)
Sikivie P., Yang Q., 2009, Phys.Rev.Lett. **103**, 111301 (2009)
Silverman M.P., Mallet R.L., Gen.Rel.Grav. **34**, 633 (2002)